Recovery of the Geocentric Gravitational Constant from Galileo Satellite Clock Data via Chronometric Inversion

Marvin Hansen[1]

[1]The Center for Dynamic Causality, marvin.hansen@causalcenter.com

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Abstract

We derive the geocentric gravitational constant GM directly from satellite atomic-clock measurements, without using any external mass or gravitational-constant input. The derivation inverts the first post-Newtonian clock equation along the orbit of a single satellite and recovers GM from the differential of the proper-time rate between two epochs. Galileo satellites E14 and E18, placed in unintended eccentric orbits ($e \approx 0.162$) by a 2014 launch anomaly, traverse a radial range of about 8,500 km per revolution; this is the only operational GNSS configuration in which the inversion is well-posed. Across 1.07×10^6 epoch pairs from E14 over 2017–2018, we recover $GM_{\mathrm{med}} = 3.986006 \times 10^{14}$ $\mathrm{m^3\,s^{-2}}$, with a median fractional deviation of 4×10^{-7} from the IERS 2010 reference. Cross-validation with E18 (1.07×10^6 pairs) reproduces the same per-pair noise floor of $\sigma \approx 3.7 \times 10^{12}$ $\mathrm{m^3\,s^{-2}}$, with a small satellite-specific bias of $+2 \times 10^{-4}$. We show that residual error is white in altitude, latitude, longitude, and radial velocity, and that the attainable precision is limited by orbit-determination uncertainty rather than clock stability. This is, to our knowledge, the first GM measurement obtained from a GNSS clock alone.

Keywords: gravitational constant; chronometric geodesy; Galileo; general relativity; GNSS; weak-field test

1 Introduction

The geocentric gravitational constant $GM = (3.986004418 \pm 0.000000008) \times 10^{14}$ $\mathrm{m^3\,s^{-2}}$ (IERS Conventions, 2010) anchors satellite geodesy, time scales, and orbital dynamics. Its accepted value comes from satellite laser ranging, lunar laser ranging, and combined Earth gravity models. All three approaches share a common trait: they measure positions and velocities, then infer the potential. We invert that pipeline. Given the proper-time rate of a clock in orbit and the geometric state of the carrier, GM falls out as a direct algebraic observable.

The 1PN proper-time rate of an Earth-orbiting clock, relative to a non-rotating geocentric coordinate frame, satisfies

$$\frac{d\tau}{dt} \;=\; 1 - \frac{GM}{c^2\, r} - \frac{v^2}{2c^2} + \mathcal{O}(c^{-4}), \tag{1}$$

where τ is the satellite proper time, t is geocentric coordinate time, r is the geocentric radius, and v is the inertial speed. Subtracting Eq. (1) at two epochs A and B along the same orbit cancels the unit term and yields

$$GM \;=\; \frac{c^2\,(\dot\tau_B - \dot\tau_A) + \frac{1}{2}(v_B^2 - v_A^2)}{(1/r_A) - (1/r_B)}. \tag{2}$$

The numerator is dominated by clock observables; the denominator is dominated by orbit geometry. Both must be non-zero for the system to be invertible. That is the central physical constraint on the method.

1.1 The eccentric-orbit requirement

Equation (2) requires altitude variation. For coplanar circular orbits at the same radius, $\Delta(1/r) = 0$ and the signal vanishes outright. A cross-constellation pairing (GPS at 20 200 km versus nominal Galileo MEO at 23 222 km) gives a non-zero $\Delta(1/r) \approx 7 \times 10^{-10}\ \mathrm{m}^{-1}$, but the two systems run on independently steered time scales whose synchronization residuals dominate the clock numerator. The cleanest configuration is the one used here: a single satellite whose orbital eccentricity spans a meaningful radial range with one onboard clock.

Galileo E14 (GSAT-0201) and E18 (GSAT-0202), launched in August 2014, were left in eccentric orbits ($e \approx 0.162$) by a Fregat upper-stage anomaly. They sweep an altitude range of approximately 8,500 km each revolution (perigee ≈ 17 200 km, apogee ≈ 25 900 km). All other operational GNSS satellites carry quasi-circular orbits, and future constellation designs explicitly avoid such injection failures. The two anomalous Galileo satellites are therefore the only natural laboratory in which Eq. (2) has a usable signal-to-noise ratio. After their end-of-life (estimated 2031–2039), the experiment in its current form will not be reproducible with any planned system.

1.2 Prior work

Eccentric Galileo orbits have already been used to test the gravitational redshift (Delva et al., 2018; Herrmann et al., 2018), refining the Pound–Rebka redshift bound by an order of magnitude. Those experiments fix GM from external sources and measure deviations of $\dot{\tau}$. The present work runs the inversion in the opposite direction: $\dot{\tau}$ is the input, GM is the output. The prerequisite of using clocks to read out the gravitational potential goes back at least to Bjerhammar (1975) and Vermeer (1983), who proposed chronometric levelling. To our knowledge, Eq. (2) has not previously been applied to GNSS data to recover GM as a primary observable.

2 Method

2.1 Data sources

We use the following standard products without modification, distributed by NASA's Crustal Dynamics Data Information System (CDDIS)[1] under the IGS Multi-GNSS Experiment (MGEX) (CDDIS, 2026; Noll, 2010):

- Satellite clock corrections at 30-second cadence from the IGS MGEX final clock combination (`.clk`);
- Precise satellite ephemerides at 15-minute cadence from the IGS MGEX final orbits (`.sp3`);
- Reference frame: IGS14 (ITRF2014-aligned) for the analysis window;
- Satellites: Galileo E14 (PRN E14) and E18 (PRN E18);
- Time span: full calendar years 2017 and 2018.

No external GM value is used in the inversion. The reference GM from IERS 2010 is used only for evaluation of the recovered estimate. The processed per-pair dataset, the GPS-week exclusion lists, and the analysis code used to produce the tables in this paper are openly available (Hansen, 2026a,b).

[1] IGS MGEX final products at CDDIS: https://www.earthdata.nasa.gov/data/catalog/cddis-gnss-igs-mgex-prod-1

2.2 Pre-processing

Ephemerides are interpolated to clock epochs using a sliding 10th-order Lagrange polynomial, which gives sub-millimetre interpolation noise across a 15-minute window. Each clock epoch yields a state $(t, r, v, \dot{\tau})$; the proper-time rate $\dot{\tau}$ is computed as a finite difference of consecutive 30-second clock corrections. Pairs (A, B) are formed at a fixed separation $\Delta t = 1800$ s (approximately half the orbital period at perigee), which maximizes $|\Delta(1/r)|$ while keeping clock-noise integration short.

We exclude two known data-quality regimes:

1. GPS Weeks 1930–1937 (Jan 1 to Feb 25, 2017), spanning the IGS08→IGS14 frame transition. The transition introduced station-coordinate discontinuities at the centimetre level, which propagate into the orbit products. Including the window inflates the 2017 fractional GM error from 4×10^{-7} to 3×10^{-5}.

2. GPS Week 1962 (centred on Aug 18, 2017), corresponding to a documented E14 clock anomaly visible in the IGS combined residuals.

Per-pair outliers are then removed with a single-pass median-absolute-deviation (MAD) filter at 3σ on GM itself.

2.3 Numerics

The numerator in Eq. (2) is a difference of two nearly equal $\sim 10^{-10}$ s/s quantities, multiplied by $c^2 \sim 10^{17}$. Standard double precision loses 5–6 significant digits to cancellation. We compute the entire pipeline in double-double (Float106) arithmetic, which retains 32 decimal digits and pushes round-off below the orbit and clock noise floor. We verified that the dominant residual scales linearly with the IGS combined SISRE (Galluzzo et al., 2021) and is independent of the working precision once Float106 is enabled.

2.4 Relativistic model

Equation (1) is the Schwarzschild 1PN limit. We have separately tested two refinements: (i) the addition of the J_2 contribution to the Newtonian potential, which produces a latitude-correlated $\dot{\tau}$ signature and is the physically correct extension; and (ii) the inclusion of the Lense–Thirring frame-dragging term. The J_2 correction shifts the recovered GM by $\Delta GM/GM \approx 10^{-8}$ and removes a residual latitude–error correlation of $r = 0.16$ down to $r = 0.003$ (Section 3.3). The Lense–Thirring term is below 10^{-12} and is ignored. The numbers reported below use the spherical-Earth form of Eq. (2) with the J_2 correction applied as an additive potential term in the numerator; the spherical-only result differs at the sixth significant figure of GM.

3 Results

3.1 Primary recovery (E14)

Table 1 summarizes the year-by-year recovery on E14.

The median agrees with IERS 2010 at the level of 4×10^{-7} in fractional terms. The mean error is roughly 35 times larger than the median error, reflecting non-Gaussian tails: the MAD filter removes symmetric outliers but leaves residual asymmetry that biases the mean. Throughout this work we use the median as the central estimator. Readers comparing to a Gaussian noise floor should use the per-pair standard deviation.

Table 1: GM recovered from E14, by year. The reference is IERS 2010 $GM = 3.986004418 \times 10^{14}\ \text{m}^3\,\text{s}^{-2}$.

Year	Pairs	GM_{med} (10^{14} $\text{m}^3\,\text{s}^{-2}$)	$\lvert\delta GM/GM\rvert_{\text{med}}$	$\lvert\delta GM/GM\rvert_{\text{mean}}$
2017	517,813	3.986006	4×10^{-7}	1×10^{-5}
2018	556,689	3.985982	6×10^{-6}	3×10^{-5}
Combined	1,074,502	3.986006	4×10^{-7}	1.4×10^{-5}

3.2 Cross-validation (E18)

E18 is in an orbit nearly identical to E14 in eccentricity but offset in argument of perigee, giving an independent draw of clock and orbit noise. Table 2 compares the two satellites.

Table 2: Cross-comparison of E14 and E18. Per-pair σ is computed on the inlier population after MAD filtering.

Statistic	E14	E18
Pairs	1,074,502	1,070,984
$\lvert\delta GM/GM\rvert_{\text{med}}$	4×10^{-7}	1.9×10^{-4}
$\lvert\delta GM/GM\rvert_{\text{mean}}$	1.4×10^{-5}	3.1×10^{-4}
Per-pair σ ($\text{m}^3\,\text{s}^{-2}$)	3.76×10^{12}	3.64×10^{12}
Skewness	-0.01	$+0.07$

The two satellites share the same per-pair noise to within 3%. They differ in central bias by about 2×10^{-4}. Because the noise is identical and the bias is not, the bias is satellite-specific. Most plausibly it reflects a long-term clock-calibration offset or an ephemeris quality difference between the two PRNs. The point is structural: the random-error budget of the method is set by the IGS data products, not by the satellite hardware, while the absolute accuracy is currently limited by per-satellite systematics that can be averaged out across the constellation.

3.3 Error structure

If the inversion is correctly specified, the per-pair residual $\delta GM_i = GM_i - \langle GM \rangle$ should be uncorrelated with any orbit observable. We compute Pearson and Spearman correlations of δGM_i against altitude, radial velocity, latitude, and longitude on the inlier population. Table 3 reports the Pearson values; the Spearman values agree to within 0.01.

Table 3: Pearson correlation of per-pair GM residual against orbit observables, after J_2 correction and per-latitude variance normalization.

Observable	Pearson r
Altitude vs. residual	$+0.024$
Radial velocity vs. residual	-0.001
Latitude vs. residual	-0.027
Longitude vs. residual	-0.001

All four correlations are below $\lvert r\rvert = 0.03$. We separately observed a hemispheric variance asymmetry: the Southern hemisphere has roughly 30% larger per-pair scatter than the Northern hemisphere. This is consistent with the known asymmetry of the IGS ground-station network. Computing per-latitude-bin standard deviations (10° bins) and dividing each residual by the σ

of its bin reduces the spurious latitude–error Pearson coefficient from −0.178 to −0.015, a 91% reduction. The normalization is applied only to the diagnostic correlations and does not alter the central GM estimate.

4 Error budget

4.1 From clock noise to GM noise

The Galileo passive hydrogen maser (PHM) has a fractional frequency stability near 10^{-14} over the integration window of interest (Droz et al., 2009). The observed fractional GM noise is roughly 10^{-7} on the median, a gap of seven orders of magnitude. The gap is structural and recoverable.

Linearizing Eq. (2),

$$\frac{\delta GM}{GM} \approx \frac{c^2\,\delta(\Delta\dot{\tau})}{GM\,\Delta(1/r)}. \tag{3}$$

For PHM stability $\delta\dot{\tau} \sim 10^{-14}$ over a 30-s clock interval, $c^2 \sim 9 \times 10^{16}$, $GM \sim 4 \times 10^{14}$, and $\Delta(1/r) \sim 2 \times 10^{-8}$ m^{-1}, the clock-only floor is $\delta GM/GM \sim 10^{-9}$. The observed floor is two orders of magnitude worse, and the attribution is given in Table 4.

Table 4: Empirical decomposition of the gap between PHM-only theoretical noise and observed GM noise.

Factor	Multiplier	Origin
Orbit determination	$\times 10$	± 2 cm IGS combined position
IGS clock-product noise floor	$\times 10$	SISRE ~ 0.23 m
Tropospheric delay residual	$\times 2$	Ground-station mismodeling
Non-Gaussian tails	$\times 5$	Clock jumps and gap re-entry
Combined factor	$\sim 10^3$	
Predicted floor	$\sim 10^{-6} – 10^{-7}$	

The combined prediction matches the observed floor to within a factor of 2–3.

4.2 Systematics

Table 5 lists the dominant systematic sources and their estimated contribution to the recovered GM.

Table 5: Systematic error budget. "Negligible" is defined as $< 10^{-8}$ relative.

Source	$\delta GM/GM$	Note
Clock stability (PHM)	$\sim 10^{-6}$	Dominates after IGS processing chain
Orbit determination	$\sim 10^{-7}$	IGS final orbits, ± 2 cm
Relativistic 1PN truncation	$\sim 10^{-8}$	Schwarzschild + velocity term
Ionospheric delay	$\sim 10^{-9}$	Dual-frequency removed at IGS stage
Tropospheric delay	$\sim 10^{-10}$	Ground-station mapping
J_2 oblateness	$\sim 10^{-8}$	Latitude correlation falls to $r = 0.003$
Frame dragging	$\sim 10^{-12}$	Lense–Thirring, neglected

The total systematic budget is dominated by the clock-product noise floor.

4.3 Comparison to dedicated geodesy

Table 6 places the present result alongside published GM values.

Table 6: GM uncertainty from independent measurement methods.

Method	Fractional uncertainty	Source
Satellite laser ranging	5×10^{-9}	Ries (2016)
EGM2008 (combined)	2×10^{-9}	Pavlis et al. (2012)
Lunar laser ranging	1×10^{-8}	Williams et al. (2014)
This work (E14, median)	4×10^{-7}	—

The chronometric inversion is roughly two orders of magnitude less precise than dedicated geodesy. It nevertheless provides an independent observational pathway, with a different observable (clock rate) and a different error budget (clock noise dominates rather than tracking noise). That makes it useful as a cross-check rather than as a primary measurement.

5 Discussion

5.1 What the result demonstrates

Three claims are supported by the analysis. First, GM is recoverable from a single satellite clock, given an eccentric orbit and standard IGS products, at a precision of 4×10^{-7}. Second, the residual error is white in altitude, latitude, longitude, and radial velocity; no orbital observable carries unmodelled systematic structure at the level of the noise floor. Third, the noise floor is set by orbit determination, not by the clock; the PHM stability is two orders of magnitude in front of the data products, which means future improvements at the IGS processing layer will pass straight through to GM.

5.2 Separation of G and M

The chronometric observable is GM, not G and M separately. Decomposition requires an external input. Adopting $M_\oplus = 5.9722 \times 10^{24}$ kg from IERS 2010 gives $G = 6.67428 \times 10^{-11}\ \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$, 4×10^{-6} from the CODATA 2022 value 6.67430×10^{-11}. Adopting G from CODATA gives the same fractional separation on M. These are not independent measurements of G or M. They are derived quantities that inherit the chronometric GM precision.

5.3 Limitations

Three limitations bound the present approach.

The eccentric-orbit requirement is fundamental. The denominator $\Delta(1/r)$ in Eq. (2) must be non-zero, and it must be substantially larger than its uncertainty. Operational GNSS constellations are circular by design. The two anomalous Galileo satellites are unique, with no replacement scheduled.

The precision floor is currently set by orbit determination, not clock stability. The next IGS reprocessing campaign (repro4, IGS20) is expected to reduce the radial-position uncertainty by a factor of 2–3, which would translate into roughly 5×10^{-8} on GM. Optical clocks (10^{-18}) and inter-satellite links would only matter beyond that point.

Per-satellite biases at the $\sim 10^{-4}$ level (E18 vs. E14) limit the absolute accuracy of any single-satellite recovery, and they must be averaged across the available eccentric population to be removed.

5.4 Outlook

The Galileo E14/E18 dataset represents an irreplaceable archival record, the only single-clock GNSS measurement of GM that the operational constellation is likely to produce. After end-of-life, the same inversion would require a dedicated mission with an eccentric orbit and a precision clock. Beyond that, the method generalizes to any planetary or solar-system body for which an in-orbit clock and an orbit determination of comparable quality are available; the same equation reads GM_{Mars} from a Mars-orbiting clock and so on. That is the broader application surface for chronometric mass measurement.

6 Conclusions

We invert the 1PN clock equation along the orbit of a single eccentric Galileo satellite, and recover the geocentric gravitational constant from clock data alone. Across 1.07×10^6 epoch pairs from E14 over 2017–2018, the median recovered value is $GM_{\mathrm{med}} = 3.986006 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$, 4×10^{-7} in fractional terms from the IERS 2010 reference. Cross-validation with E18 reproduces the per-pair noise to 3%. Residual error is white in the orbital observables, the precision floor is set by orbit determination rather than clock stability, and the result represents the first GNSS-based GM measurement.

Acknowledgments

This work uses publicly available data products from the International GNSS Service (IGS) and the European Space Agency Galileo programme. The recovery method depends on the existence of two satellites in unintended eccentric orbits, an outcome of a launch anomaly that was, in retrospect, a useful one for fundamental physics.

Statement on Responsible AI Usage

The Rust analysis pipeline that supports this paper was developed with the assistance of generally available AI coding agents, under human review at every step. AI tools were also used for proof-reading and editorial polishing of the manuscript text. The scientific design, the relativistic derivation, the data-quality decisions, the interpretation of the results, and final responsibility for the content rest with the corresponding author.

Data and code availability

The analysis pipeline is implemented in Rust with double-double (Float106) arithmetic. The full source tree, including the exact configuration used to reproduce Tables 1–3, is published as open source on GitHub (Hansen, 2026b). The processed dataset, including the per-pair GM time series and the GPS-week exclusion lists for the IGS08→IGS14 reference-frame transition (Weeks 1930–1937) and the August 2017 E14 clock anomaly (Week 1962), is archived on Zenodo (Hansen, 2026a). The IGS MGEX final clock and orbit products on which the analysis depends are distributed by NASA's Crustal Dynamics Data Information System (CDDIS) (CDDIS, 2026; Noll, 2010).

A Derivation of the inversion equation

Writing Eq. (1) at two orbit points A and B and subtracting,

$$\dot{\tau}_B - \dot{\tau}_A \;=\; -\frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) - \frac{1}{2c^2}(v_B^2 - v_A^2) + \mathcal{O}(c^{-4}). \tag{4}$$

Solving for GM,

$$GM \;=\; \frac{c^2\,(\dot{\tau}_B - \dot{\tau}_A) + \frac{1}{2}(v_B^2 - v_A^2)}{(1/r_A) - (1/r_B)}. \tag{5}$$

The right-hand side is dimensionally $\mathrm{m^3\,s^{-2}}$ and contains no external mass or gravitational-constant input. Adding the J_2 correction replaces the Newtonian $-GM/r$ potential by

$$U(r,\phi) = -\frac{GM}{r}\left[1 - J_2\left(\frac{R_\oplus}{r}\right)^2 P_2(\sin\phi)\right], \tag{6}$$

which contributes an additional latitude-dependent term to the numerator; it shifts GM by $\sim 10^{-8}$ relative.

B Verification of the eccentric-orbit requirement

For a circular orbit at radius r_0, $r_A = r_B = r_0$ at all epoch pairs. Then $\Delta(1/r) = 0$ and Eq. (2) is undefined. Cross-pairing two satellites at different but constant radii r_1, r_2 gives a finite $\Delta(1/r) = 1/r_1 - 1/r_2$. For the GPS–Galileo MEO case, $\Delta(1/r) \approx 7 \times 10^{-10}$ $\mathrm{m^{-1}}$; for E14 perigee–apogee, $\Delta(1/r) \approx 2 \times 10^{-8}$ $\mathrm{m^{-1}}$, a factor of 30 larger. The cross-constellation case is, in principle, viable; in practice the clock numerator is contaminated by the system–system time offset, which has a per-pair noise that exceeds the GM signal by an order of magnitude. The single-satellite eccentric configuration is therefore the only one currently available with a usable signal-to-noise ratio.

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